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FURTHER PERCENTAGE POINTS FOR GREENWOOD'S STATISTIC.(U)
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By

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FURTHER PERCENTAGE POINTS FOR GREENWOOD'S STATISTIC

By

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1. INTRODUCTION

There has recently been a revival of interest in Greenwood's (1946) statistic, labeled G below, and its use in testing for uniformity of points on a line. Burrows (1979) has published percentage points for G for this test, for samples of size n up to 10. In this paper we extend Burrows' tables, and add some brief comments on recent works on the effectiveness of G and related statistics as test statistics for uniformity.

Greenwood's statistic G is calculated from a sample of n values in the interval $(0,1)$ as follows. Let $x_1, x_2, \ldots, x_n$ be the sample values, in ascending order, and define d_i to be the spacing $d_i = x_i - x_{i-1}$, $i = 2,\ldots,n$; let $d_1 = x_1$, and $d_{n+1} = 1-x_n$. Greenwood's statistic is then

$$G = \sum_{i=1}^{n+1} d_i^2 .$$

Suppose H_0 is the null hypothesis that the x_i are, before being ordered, a random sample from the uniform distribution with limits 0, and 1. The statistic G is a natural statistic for testing H_0, however, the distribution of G, for H_0, is difficult to find, even for small samples.

Moran (1947) gives many results on G, including the moments for H_0, and both Moran and Burrows (1979) have references to early work on the distribution theory of G. Moran showed by means of the moments that G has an asymptotic normal distribution, but the rate of convergence to normality is so slow that this result cannot be used to give percentage points for test purposes. Burrows gives points for $n \leq 10$, and these are derived from a recursion relation. To these we now add, in Table 1 points for $n > 10$, for nG rather than G. These were found by fitting Pearson curves to the moments. This technique will give very accurate results (Solomon and Stephens, 1978), and we verify this by comparing points for $n = 10$ with those of Burrows (1979). The statistic nG is tabulated rather than G to make interpolation easier.

Hill (1979) has given an algorithm also using four moments, to approximate the percentage points by Johnson curves. He compares the approximate points for $n = 10$ with Burrows' exact points and finds good agreement. Unfortunately the points given by Hill are incorrect; the algorithm reproduces a misprint in the moments given by Moran (1947). The third moment about the mean, μ_3, has numerator $8n(10n-4)$ and not $8(10n^2-4)$ as given by Moran and used by Hill. The error and correction have been recently confirmed by a private communication from Professor Moran* and of course it is easy to correct

* This was also pointed out by Hartley and Pfaffenberger (1972).

the Hill algorithm. When corrected, the plot of β_1, β_2 follows a curve somewhat like that drawn by Hill, but with smaller values of β_1; some values are given in Table 2. The Johnson curves, as before, will change character near $n = 12$, and the reservations expressed by Hill would still apply.

Hill also mentions the fact that use of the lower endpoint of the distribution (the smallest value of G is $1/(n+1)$, when all $d_i = 1/(n+1)$) gave worse results than using four moments only. This difficulty might well disappear with the correction. In our experience, use of three moments and the lower endpoint gave improved values in the lower tail, but worse in the upper tail; this would be expected, since the lower tail is very steep for small n (see Burrows' Figure 1) and use of the correct endpoint will be a great advantage. However, as n increases the difference in percentage points obtained by the two methods is smaller; more importantly, the upper tail points are those most likely to be needed in practice, so for Table 1 it was decided to use a four moment fit. This fit, for $n = 10$, gives lower and upper 1% and 5% points for $10G$ equal to 1.154, 1.222, 2.412 and 2.997 to be compared with Burrows' exact values of 1.117, 1.211, 2.404, and 3.008. Except for the lower 1% point, there will be negligible error in significance level obtained by using the Pearson curve points for this value of n, and the accuracy can be expected to improve as n becomes larger. Using three moments and the lower endpoint the points are 1.116, 1.207, 2.399, 2.987. (Incidentally, the points given by Hill for $n = 10$ are marginally better than the Pearson curve 4-moment fit, thus demonstrating that it is

sometimes better to be approximately wrong than to be approximately right. For $n = 5$, however, the Pearson curve points are definitely better).

It will be interesting to see a comparison of Johnson curve values with the Pearson curve values; previous experience has often shown that they are very close together. However, the author does not have the Johnson curve algorithm, and has in fact had the Pearson curve algorithm for several years (they have been used in comparing various goodness-of-fit procedures) so it seems worthwhile to present them now. The algorithm used is being prepared for publication. It involves interpolation in the extensive tables of Pearson curves given in Biometrika Tables, Vol. 2. Computer routines are also available (Solomon and Stephens, 1978).

2. COMMENTS ON GOODNESS-OF-FIT

Over the years, there has been a steady interest in statistics based on spacings in general, and Pyke (1965) gives a very full discussion of work to that date. More recently, there has developed new interest in G itself or statistics similar to G. Hartley and Pfaffenberger (1972) introduced a statistic S^2 which is related to G by $S^2 = \{(n+1)G-1\}(n+2)$, so that it is effectively the same as G for testing purposes. The author, in an unpublished Technical Report (Stephens, 1974) made some comparisons of S^2 with EDF (empirical distribution function) statistics D, W^2, U^2 and A^2, in tests for uniformity and concluded that S^2 (i.e., G) has power somewhere between U^2 and A^2, depending on the alternative. The points for

S^2 were derived from the points now given for G. Further comparisons have been made by Quesenberry and Miller (1977) and the author has also continued a study of tests for uniformity, the results to be published elsewhere. Hartely and Pfaffenberger (1972) and del Pino (1978) have also discussed k-spacings, i.e. spacings between the ordered x_i, taken k at a time. Pyke (1965) refers to doubts that spacings provide effective statistics for tests of uniformity, but these issues are not yet clear; see del Pino (1978) for most recent work on these lines, and for other references. Spacings have a natural appeal, for example, when they arise as the intervals between events formed by a renewal process, and it is hoped that provision of percentage points for G or for nG will encourage further work on the properties of this statistic.

TABLE 1

Upper and lower percentage points for nG

Sample size	Percentage level α							
	Lower tail				Upper tail			
n	.01	.025	.05	.10	.10	.05	.025	.01
12	1.198	1.234	1.272	1.326	2.204	2.441	2.683	3.015
14	1.233	1.272	1.312	1.368	2.227	2.457	2.691	3.014
16	1.263	1.304	1.346	1.403	2.242	2.464	2.691	3.003
18	1.288	1.332	1.375	1.433	2.251	2.466	2.685	2.988
20	1.311	1.356	1.400	1.459	2.258	2.465	2.677	2.970
25	1.358	1.405	1.451	1.510	2.265	2.456	2.651	2.920
30	1.395	1.444	1.490	1.549	2.265	2.443	2.624	2.873
40	1.453	1.502	1.548	1.605	2.258	2.415	2.573	2.790
50	1.495	1.544	1.589	1.644	2.248	2.389	2.531	2.723
60	1.529	1.577	1.621	1.674	2.238	2.367	2.495	2.669
80	1.579	1.625	1.666	1.716	2.220	2.331	2.441	2.587
100	1.616	1.659	1.698	1.745	2.205	2.304	2.400	2.528
200	1.714	1.750	1.781	1.818	2.159	2.226	2.289	2.371
500	1.811	1.836	1.858	1.884	2.107	2.147	2.183	2.228

TABLE 2

Values of moment parameters β_1 and β_2 for selected sample sizes n.

n	5	6	8	10	50	100	500
β_1	2.518	2.700	2.877	2.912	1.484	0.858	0.194
β_2	6.827	7.358	8.023	8.351	6.378	5.026	3.473

TABLE 3

Comparisons of exact points for G with various approximations.

PC(4) and PC(3) refer to Pearson curve approximations using 4 moments or 3 moments and the lower end point.

n = 5	α:	.01	.05	.10	.90	.95	.99
Exact		.1839	.1994	.2101	.3830	.4320	.5475
Hill		.1988	.2074	.2144	.3860	.4377	.5505
PC(4)		.1942	.2026	.2104	.3856	.4330	.5401
PC(3)		.1837	.1979	.2085	.3831	.4292	.5381
n = 10							
Exact		.1116	.1211	.1272	.2157	.2404	.3008
Hill		.1138	.1220	.1276	.2161	.2406	.3007
PC(4)		.1154	.1222	.1274	.2168	.2412	.2997
PC(3)		.1116	.1207	.1268	.2160	.2399	.2987

REFERENCES

Burrows, P.M., (1979). Selected percentage points of Greenwood's statistic. J.R. Statist. Soc. A, 142, 256-258.

del Pino, Guido E. (1979). On the asymptotic distribution of k-spacings with applications to goodness-of-fit tests. Ann. Statist., 7, 1058-1065.

Greenwood, M. (1946). The statistical study of infectious diseases. J.R. Statist. Soc. A, 109, 85-109.

Hartley, H.O. & Pfaffenberger (1977). Quadratic forms in order statistics used as goodness-of-fit criteria. Biometrika, 59, 3, 605-611.

Hill, I.D. (1979). Approximating the distribution of Greenwood's statistic with Johnson distribution. J.R Statist. Soc. A, 142, 378-380.

Moran, P.A.P. (1947). The random division of an interval. J.R. Statist. Soc. B, 9, 92-98.

Pyke, R. (1965). Spacings. J.R. Statist. Soc. B, 27, 395-449.

Quesenberry, C.P. & Miller, F.L. (1977). Power studies of some tests for uniformity. J. Statist. Comput. and Simul., 5, 169-191.

Solomon, H. & Stephens, M.A. (1978). The use of Pearson curves as approximations to densities. Jour. Amer. Statist. Assn., 73, 153-160.

Stephens, M.A. (1974). On a test statistic for uniformity based on spacings. Technical Report, Department of Statistics, Stanford University.

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Percentage points for Greenwood's statistic obtained by fitting Pearson curves to the first four moments are given. Comparisons are given with the exact points for $n = 10$, recently given by Burrows (1979) and these suggest that the approximate points will be accurate for practical purposes.

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